arXiv:hep-th/0311063v2 14 Nov 2003

# Radiative corrections in 5D and 6D expanding in winding modes

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1st February 2008

**Abstract**

We compute radiative corrections in five and six dimensional field theories, using winding modes in mixed momentum-coordinate space. This method provides a simple way of finding UV divergencies, finite corrections and localized terms when the space is compactified on orbifolds. As an application we compute the finite piece of scalar masses, the logarithmic contributions to the couplings and the effect of localized parallel and perpendicular kinetic terms. We apply it to get a two loop effective potential that can stabilize large extra dimensions.

UAB-FT-553

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# 1 Introduction

The Standard Model is not a fundamental theory and there have been many proposals to go beyond it. It is expected that a field theory with extra dimensions arises as the low energy limit of a fundamental string theory. For this reason extra dimensions are a common feature of any theory valid at high energies. Almost all problems of particle physics have been reformulated in this context giving new possibilities. In particular, physics of large extra dimensions can provide solutions to the hierarchy problem [1], [2] and new mechanism of symmetry breaking [3], that can be tested in the next high energy experiments.

In this note we developed a formalism that allows us to compute loop corrections in field theories with large extra dimensions, separating UV divergencies from finite contributions, in a direct product space. Usually, higher dimensional theories are formulated using Kaluza-Klein decomposition. Instead in this work we will use winding modes. In a five dimensional theory these modes are obtained propagating around the circle of the extra dimension. Two paths with different windings are topologically different. Ultraviolet divergencies in loop integrals are associated to the zero winding mode. Non zero modes are long distance and each of them will give finite terms [4]. Then this formalism is very useful to separate finite from divergent contributions.

We will use winding decomposition to compute radiative corrections on 5D and 6D theories. When the theory is compactified on an orbifold translation symmetry is broken on the fixed points; we will show how localized terms are generated at one loop. We also analyze the role of localized kinetic terms, parallel and perpendicular to the brane, interpreting them in a very intuitive way, and showing their physical effects. We show these terms can provide new mechanisms of symmetry breaking. We will compute general finite masses and also self and gauge couplings using this method.

When there are more than one extra dimensions, winding modes prove to be very useful. In particular, we consider a 6D theory and after obtaining the propagator in mixed representation, we use it to compute radiative corrections, the finite piece of the masses and the couplings.

As an interesting application we will apply this formalism to compute a two loop effective potential. We will prove that this potential can stabilize large extra dimensions when there are brane terms.

In section 2 we define winding modes working on a mixed momentum-coordinate representation. We apply this idea to a 5D theory in section 3. In section 4 we show how to work with more than one extra dimension. In section 5 we compute a two loop effective potential and section 6 is for conclusions.

During the writing of this work another paper appeared [5] on similar subjects, reaching the same results about mixed momentum-coordinate representation and mass terms.

# 2 Winding modes

Let's consider a 5D theory compactified on $\mathcal{M}^4 \times \mathcal{C}^1$, where $\mathcal{M}^4$ is 4D Minkowsky space and $\mathcal{C}^1$ is a compact 1-D manifold. If we can write $\mathcal{C}^1 = \mathcal{R}^1/G$, where $\mathcal{R}^1$ is the real line and $G$ is a discrete group acting freely on $\mathcal{R}^1$, then we can define winding modes. The simplest example

is $\mathcal{C}^1 = S^1$ (a circle) and $G = \mathcal{Z}$ (the set of integer numbers, with the sum defined as the group product). In this case we obtain the compact space identifying $y \sim y + n2\pi R$. Due to the identification $y \in [0, 2\pi R)$, the index $n$ labels winding modes.

This idea suggests the following procedure: we compute on the infinite space and identify $y \sim y + n2\pi R$ to obtain the physical magnitudes on the compact space. For a general compact space $\mathcal{C}^m = M^m/G$ of dimension $m$ ($M^m$ a non compact m-dimensional manifold), we identify $z \sim g(z)$, $z \in \mathcal{C}^m$ and $g \in G$. Then we can define winding modes for every $g$, and associate divergencies to the zero mode $g = 1$.

To show how this formalism works we will compute the massless scalar propagator on Euclidean 5D spacetime. We work on a mixed representation $(p_\mu, y)$, where $p_\mu$ is the 4D momentum and $y$ the coordinate in the extra dimension. Then we can obtain the Green's function from the following equation

$$(p^2 - \partial_y^2)\widetilde{G}(p; y - y') = \delta(y - y'), \tag{1}$$

where we have Fourier transform to momentum space only on the 4D space, and $p^2 = p^\mu p_\mu$. Solving this equation we get

$$\widetilde{G}(p; y - y') = \frac{e^{-p|y-y'|}}{2p}. \tag{2}$$

Due to translation invariance we see that the propagator only depends on $|y-y'|$. If we consider a massive scalar field we just have to replace $p^2 \to p^2 + m^2$.

To get the propagator on the compact space we identify $|y - y'| \sim |y - y' + 2n\pi R|$ (see fig. 1). Then we restrict $y, y' \in [0, 2\pi R)$ and sum over windings

$$\widetilde{G}^{cir}(p; y, y') = \sum_{n=-\infty}^{n=\infty} \widetilde{G}(p; y - y' + 2n\pi R) = \sum_n \frac{e^{-p|y-y'+2n\pi R|}}{2p}. \tag{3}$$

We can solve this sum, but it is more useful to consider the contribution of each mode. In the last equation we can see that for $n \neq 0$ the propagator is exponentially damped, therefore loop integrals over momenta will be finite. On the other hand, for $n = 0$, the propagator goes as $p^{-1}$ when evaluated in $y = y'$. For these reasons, compactifying on a circle, we will obtain divergent contributions for the winding 0-modes, and finite contributions for the other modes. Therefore, using the winding modes formalism, is very easy to separate divergent from

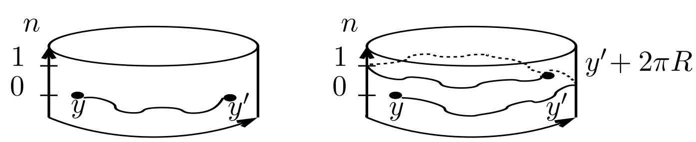



Figure 1: *Two equivalent contributions for the propagator between $y$ and $y'$. In the vertical axis we represent the number of windings.*

finite contributions. The divergent terms are associated to short distances, that is why the divergencies resulting from the 0-mode are the same as the divergencies of the uncompactified theory. For the masses, the radiative corrections are dominated by high energy effects. High energy implies small distances, then for the masses, the small $n$ contributions dominate over

large $n$. This is not the case when we do Kaluza-Klein decomposition. This is one of the advantages of winding decomposition.

**Orbifold compactification**

Orbifolds are used to obtain 4D chiral fermions from a higher dimensional theory. In general we can obtain an orbifold with a discrete group $F$ acting non freely on the compact space $\mathcal{C}$. Points of $\mathcal{C}$ left invariant by $F$ are fixed points, and there $\mathcal{C}/F$ is singular. The simplest example is the orbifold $S^1/Z_2$, where $Z_2 : y \to -y$ is the parity transformation on the extra dimension. Due to the identification $y \in [0, \pi R]$ and the fixed points are $y = 0, \pi R$. To give a complete description we also have to specify field parities.

We first consider a scalar field on $S^1/Z_2$, with definite parity $Z_2\phi(x^\mu, y) = \phi(x^\mu, -y) = \pm\phi(x^\mu, y)$. Due to the identification $y \sim -y$, we can propagate from $y$ to $y'$ and from $y$ to $-y'$, then the propagator is [6]

$$\widetilde{G}^{orb}_{\pm}(p; y, y') = \sum_n \left(\frac{e^{-p|y-y'+2n\pi R|}}{2p} \pm \frac{e^{-p|y+y'+2n\pi R|}}{2p}\right), \tag{4}$$

where $y, y' \in [0, \pi R]$, and $\pm$ depends on the field parity. This propagator depends on $(y + y')$ due to the breaking of translation invariance.

In this equation we can see that the propagator goes as $p^{-1}$ for the following limits of the coordinates and windings: $(y \to 0, y' \to 0, n = 0)$ and $(y \to \pi R, y' \to \pi R, n = -1)$. Then we expect divergencies localized on the fixed points of the orbifold [7]. We can expect this terms because they don't break any symmetry of the theory.

# 3 5D radiative corrections on $\lambda\phi^4$

We compute one-loop radiative corrections of the scalar interacting theory defined by

$$S = \int d^4x\, dy\, [\frac{1}{2}(\partial_M \phi)^2 - \frac{\lambda}{4!}\phi^4]. \tag{5}$$

We will consider our toy model an effective theory valid to some cut-off $\Lambda$, that by naive dimensional analysis should be $\Lambda \sim 24\pi^3\lambda^{-1}$. Furthermore, the lagrangian in eq. (5) shall be defined as the effective theory at the scale $\Lambda$. In performing quantum corrections we will cut off the 4D momentum integral at the scale $\Lambda$. The loop contributions that depend on $\Lambda$ will signal the divergencies of the 5D theory. In fig. 2 we can see the Feynman diagrams that renormalize the two point function and the coupling at one loop. Then we can write the effective action

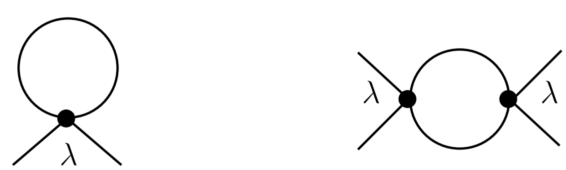



Figure 2: *Feynman diagrams for one-loop mass and vertex in the scalar interacting theory.*

with one-loop quantum corrections as

$$S_{eff} = S_{cl} + S_2 + S_4 + ... = S_{cl} - \int dy\, m^2(y)\phi^2(y) - \int dy dy' \phi^2(y)\lambda(y,y')\phi^2(y') + ... \qquad (6)$$

Let's consider first, the extra dimension compactified on a circle, then $y \in (0, 2\pi R)$. For the circle $m^2(y)$ is constant and is given by (the one-loop mass was calculated in Ref. [8] using K-K decomposition)

$$m^2_{cir} = \frac{\lambda}{2}\int \frac{d^4p}{(2\pi)^4}\left(\frac{1}{2p} + \sum_{n\neq 0}\frac{e^{-p2|n|\pi R}}{2p}\right) = \frac{\lambda}{16\pi^2}\left(\frac{\Lambda^3}{6} + \frac{1}{(2\pi R)^3}\sum_{n\neq 0}\frac{1}{|n|^3}\right), \qquad (7)$$

where we have thrown away terms that cancel when $\Lambda R \to \infty$. As we argued in the previous section, divergencies are associated to $n = 0$ and finite terms to $n \neq 0$. This expresion is valid for general $\phi(y)$, because $m^2(y)$ is independent of $y$.

If there is a symmetry (like supersymmetry or a gauge symmetry) prohibiting divergent masses, then the finite term is a prediction of the theory.

Now we compute $S_4$ at one loop. For simplicity, we will take $\phi(y) = \phi_c$ =constant. According to this, expanding in powers of external momenta we just keep the zero order terms. Then we get for $S_4$

$$S_4 = -\frac{\lambda^2\phi_c^4}{2}\int dy \int dy' \sum_{n,n'}\int \frac{d^4p}{(2\pi)^4}\widetilde{G}(p, y - y' + 2n\pi R)\widetilde{G}(p, y' - y + 2n'\pi R). \qquad (8)$$

There are two propagators involved, so there are two winding indexes. When the topology of the extra space is more complicated (for example when there are more than one extra dimension) it is useful to express this equation in terms of just one propagator as

$$S_4 = -\mathcal{I}\lambda^2\phi_c^4/2, \qquad (9)$$

where $\mathcal{I}$ is given by [1]

$$\mathcal{I} = -\int dy \sum_n \int \frac{d^4p}{(2\pi)^4}\frac{d}{dp^2}\widetilde{G}(p; y, y + 2n\pi R). \qquad (10)$$

Integrating coordinates and momenta we get the desired result, and the one-loop contribution to $\lambda(y, y')$ is

$$\lambda_{cir} = \frac{\lambda^2}{64\pi^2}\left(\Lambda - \sum_{n\neq 0}\frac{1}{|n|\pi R}\right), \qquad \phi = \phi_c. \qquad (11)$$

The sum over $n$ is logarithmically divergent in the IR, so we have to introduce an IR cut-off, that means that we sum to $n_{max} = (2\pi R\mu_{ir})^{-1}$, regulating the long distance behavior. If the field is massive the mass is the natural cut-off. The IR logarithm is the same as in 4D, and this can be easily understood in terms of K-K modes. The zero K-K mode is massless, and this is the mode that propagates long distances.

[1] To see that eq. (8) and eq. (9) are the same we can write eq. (8) in K-K modes without external momenta as $\sum_{p_5}\int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+p_5^2)^2}$, with $p_5$ the momentum in the extra dimension. The integrand can be written as $\frac{1}{(2\pi)^4}\frac{-d}{dp^2}(p^2 + p_5^2)^{-1}$, and by Fourier transformation we obtain eq. (9).

### 3.1 Radiative corrections on orbifolds

The one-loop contribution to $S_2$ on $S^1/Z_2$ are generated by the following expression (in Ref. [9] are given the masses for K-K modes)

$$S_2 = -\frac{\lambda}{2}\int_0^{\pi R} dy\,\phi^2(y)\int\frac{d^4p}{(2\pi)^4}\sum_n\left(\frac{e^{-p2|n|\pi R}}{2p}\pm\frac{e^{-p2|y+n\pi R|}}{2p}\right)\equiv S_S\pm S_Z, \tag{12}$$

where $S_S$ is the same as $S_2$ for the circle, except that $y\in(0,\pi R)$. The second term depends on $y$ and it's new. As we argued, it has divergencies for $n=0,-1$.

To obtain the divergencies we expand $\phi^2(y)$ in powers of $y$ around the fixed points $y_{fp}$. Expanding $\phi^2$ to second order, $S_Z$ is given by

$$S_Z\simeq-\frac{\lambda}{16\pi^2}\sum_{fp}\{\phi^2(y_{fp})[\sum_{n\neq0,-1}\frac{(1+2n)sgn(n)}{n^2(1+n)^2 32\pi^2R^2}+\frac{\Lambda^2}{8}-\frac{1}{16\pi^2R^2}]+\partial_y^2\phi^2(y_{fp})\frac{\log(\Lambda R)}{16}\}. \tag{13}$$

The $\Lambda^2$ divergencies are associated to $n=0$ when $y\to0$ and to $n=-1$ when $y\to\pi R$. Then these divergencies are localized on the fixed points of the orbifold. This is because the orbifold compactification breaks translation invariance in these points.

From eq. (12) we can see that $m^2(y)$ has two contributions. The first one is the same as in the circle, eq. (7). The second contribution depends on $y$ and, from eq. (13), we can see the divergent terms. We split $m^2(y)$ in divergent and finite contributions as $m^2(y)=m^2_{div}(y)+m^2_f(y)$. Then the divergent contribution is given by

$$m^2_{div\pm}(y)=\frac{\lambda}{16\pi^2}\{\frac{\Lambda^3}{6}\pm\sum_{fp}\delta_{fp}[\frac{\Lambda^2}{8}+\frac{\log(\Lambda R)}{16}\partial_y^2]\}. \tag{14}$$

The finite term can be computed for a constant field, and is given by

$$m^2_{f\pm}=\frac{\lambda}{128\pi^5R^3}[\sum_{n\neq0}\frac{1}{|n|^3}\pm\sum_{n\neq0,-1}\frac{(1+2n)sgn(n)}{2n^2(1+n)^2}\mp1],\qquad\phi=\phi_c. \tag{15}$$

To get $S_4$, we again expand the fields in powers of $y$ around the fixed points $y_{fp}$. We just take the zero order term in the power series and use eqs. (9) and (10) with the orbifold propagator. Then $S_4$ is given by

$$S_4\simeq-\frac{\lambda^2}{16\pi^2}\sum_{fp}\phi^4(y_{fp})[\frac{\Lambda\pi R}{8}+\sum_{n>1}(\frac{1}{4n}\pm\frac{1}{8}\log\frac{n+1}{n-1})\pm\frac{\log(\Lambda R)}{4}]. \tag{16}$$

The linear UV divergence is due to the zero winding mode. The logarithmic divergence is localized on the fixed points, then it is 4D, and is associated to $n=0,-1$. Therefore we can write $S_4$ as

$$S_4=-\int dy\,\lambda^\pm(y)\,\phi^4(y), \tag{17}$$

$$\lambda^\pm(y)=\lambda^\pm_f(y)+\frac{\lambda^2}{64\pi^2}\sum_{fp}\delta_{fp}[\frac{\Lambda\pi R}{2}\pm\log(\Lambda R)], \tag{18}$$

where $\lambda_f^{\pm}(y)$ is a finite coupling. For a constant field $\lambda_f^{\pm}$ is given by

$$\lambda_f^{\pm}(y) = \frac{\lambda^2}{16\pi^2}\Big[\frac{1}{\pi R}\sum_{n>1}\Big(\frac{1}{2n} \pm \frac{1}{4}\log\frac{n+1}{n-1}\Big)\Big], \qquad \phi = \phi_c. \tag{19}$$

If the field is even there are logarithmic IR divergences, as in the circle, but this doesn't happen in the odd case. This is easier to understand in terms of K-K modes: only the even field has a massless mode.

In dimensional regularization scheme, the effective scale $\Lambda$ in the 4D logarithm of the coupling, eq. (18), is replaced by an arbitrary scale $\mu$. Further discussions in this scheme can be found in Ref. [7].

## 3.2 Localized kinetic terms

We have seen that new terms localized on the branes have been induced, showing that these terms should be taken from the begining. Here we want to consider the effect of these terms on the physical parameters. We are interested in the kinetic terms: they can be parallel $\mathcal{L}^{\parallel} = ap^2\delta_{fp}\phi^2$ or perpendicular $\mathcal{L}^{\perp} = b\delta_{fp}\partial_y^2\phi^2$ to the brane (see [10] for the most general case), where $a, b$ are the couplings. Perpendicular kinetic terms generate classical divergencies, similar to classical divergencies in electromagnetism. They can be regularized with a fat brane, for example defining a $\delta_\epsilon \to \delta$ when $\epsilon \to 0$. Then, including the right counterterms, we can renormalize the theory (for classical renormalization with branes of codimension bigger than one see [11]).

The renormalization can be performed in the following way: we obtain the propagator $\widetilde{G}^{\perp}$ (with perpendicular terms) as a perturbative series with $b$-vertex insertions, as is shown in fig. 3. Every term of order $n$, has divergent contributions of order $\epsilon^{-n} \sim \Lambda_B^n$, where $\Lambda_B$ is a brane scale. To cancel the divergent terms when $\epsilon \to 0$, we have to add counterterms proportional to $\delta^n$, where $n$ is the number of vertex insertions. The divergencies can be interpreted as contributions coming from processes at energy $\Lambda_B$ on the branes. Then, including these counterterms, we are neglecting high energy contributions that can feel the brane structure. But this is exactly what we want, an approximation valid for energies $E \ll \Lambda_B$.

After that, in the theory there are only parallel terms. To see this we can resum the perturbative series (without the divergent terms) [10] and get

$$\widetilde{G}^{\perp}(p; y, y') = \widetilde{G}^{(0)}(p; y, y') - \frac{bp^2\widetilde{G}^{(0)}(p; y, y_j)\widetilde{G}^{(0)}(p; y_j, y')}{1 + bp^2\widetilde{G}^{(0)}(p; y_j, y_j)}. \tag{20}$$

where $\widetilde{G}^{(0)}$ is the free propagator. Then $\widetilde{G}^{\perp}$ is the same propagator as $\widetilde{G}^{\parallel}$ (with parallel kinetic terms) changing $b \to a$, as we will see in the following paragraphs, eq. (23). So there is nothing new considering perpendicular couplings, we can obtain all the relevant information analyzing parallel terms. For these reasons we will concentrate on parallel kinetic terms.

**Parallel kinetic terms**

Let's consider a scalar interacting theory

$$S = \int d^4x dy\Big[\frac{1}{2}(\partial_M\phi)^2 + \sum_{fp}\frac{a_{fp}}{2}\delta(y - y_{fp})(\partial_\mu\phi)^2 - \frac{\lambda}{4!}\phi^4\Big]. \tag{21}$$

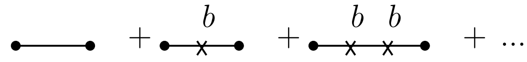
$\widetilde{G}^{\perp} =$ 

Figure 3: *Perturbative expansion of the scalar propagator with perpendicular kinetic brane terms.*

To obtain the propagator we apply the ideas of the previous sections. For the moment we suppose that the extra dimension is infinite and compute the propagator with just one delta on one of the fixed points $y = y_{fp}$. In this case the Green's function equation in mixed momentum-coordinate representation is

$$(p^2 - \partial_y^2)\widetilde{G}(p; y, y') + ap^2\delta(y - y_{fp})\widetilde{G}(p; y_{fp}, y') = \delta(y - y'). \tag{22}$$

This equation is similar to the free one but with a new source of magnitude $-ap^2\widetilde{G}(p; y_{fp}, y')$ in $y = y_{fp}$. Then the propagator is

$$\widetilde{G}(p; y, y') = \frac{e^{-p|y-y'|}}{2p} - \frac{ap}{2 + ap}\frac{e^{-p(|y-y_{fp}|+|y_{fp}-y'|)}}{2p}. \tag{23}$$

It is immediate to read the second term as a reflection of magnitude $\mathcal{X} = ap/(2 + ap)$ on the brane on $y_{fp}$. Once we realize this, we can compute the propagator in the compact space in a perturbative way (in [4] we can see the series with mass insertions ressumed, but we want to keep our intuition working with winding modes). We have to sum over all the contributions coming from reflections on $\delta(y - y_{fp} + 2n\pi R)$, in a similar way we do with light travelling in a medium with different indexes of reflection $\mathcal{X}$ and transmission $\mathcal{T} = 1 - \mathcal{X}$. A wave of amplitude $\mathcal{A}$ arrives to a $\delta$ in $n\pi R$, there $\mathcal{AX}$ is reflected and $\mathcal{AT}$ is transmited, and due to the propagation the wave amplitude is damped $e^{-pd}$ after travelling a distance $d$. With these rules we can obtain the propagator to any order in $e^{-p\pi R}$.

We will study the limit of $a \to \infty$ for fixed 4D coupling $\lambda_{4D}^R = \lambda\pi R/(\pi R + a)^2$. The propagator in the orbifold (with 4D kinetic term canonically normalized) is given by

$$\widetilde{G}_{\pm}^R(p; y, y') = \sqrt{\lambda_{4D}^R}\frac{\pi R + a}{\pi R}[\widetilde{G}(p; y, y') \pm \widetilde{G}(p; y, -y')], \tag{24}$$

where $\widetilde{G}(p; y, y')$ is given in eq. (23).

For simplicity we consider first a kinetic term on $y_{fp} = 0$. The propagator of eq. (23) has two regimes depending on wether $ap \gg 1$ or $ap \ll 1$. We define a critic winding $n_c = a(2\pi R)^{-1}$, then $|n| \ll n_c$ corresponds to the high energy regime and $|n| \gg n_c$ to the low energy. Therefore the propagator is given by

$$\begin{aligned}
\widetilde{G}_{\pm}^{n<n_c}(p; y, y') &\simeq \sqrt{\lambda_{4D}^R}\frac{\pi R + a}{\pi R}\widetilde{G}_{-}^{orb}(p; y, y'), \\
\widetilde{G}_{\pm}^{n>n_c}(p; y, y') &\simeq \sqrt{\lambda_{4D}^R}\frac{\pi R + a}{\pi R}\widetilde{G}_{\pm}^{orb}(p; y, y').
\end{aligned} \tag{25}$$

Thus we see that at short distances the field is similar to an odd field. When $n \sim n_c$ $\widetilde{G}$ is more complicated, but for a rough estimation we can consider eq. (25) valid for all $n$. It is important to note that the two pieces of equation (25) are sensible to different values of momenta.

**Effect of parallel terms on the physical parameters**

Once we have the propagator we can compute the mass and selfcoupling at one-loop with brane kinetic terms. Repeating the steps done in the previous section we calculate the one-loop mass. We consider a constant field $\phi_c$, and integrating over the extra dimension we obtain the 4D mass. There are divergent and finite contributions. Here we show the finite terms, splitted in two contributions, depending on the winding values

$$\begin{aligned} m^2_{4D\pm} = \frac{\lambda^R_{4D}}{128\pi^4 R^2}\Big[\sum_{n=1}^{n_c-1} e^{-n/nc}\frac{n^2+2nn_c+2n_c^2}{n^3 n_c^2} - e^{-1/nc}\frac{1+n_c}{n_c} + \frac{2e^{-1}}{n_c^2} \\ + \sum_{n=n_c}^{\infty}(-e^{-n/nc}\frac{n^2+2nn_c+2n_c^2}{n^3 n_c^2} + \frac{1}{n^3} \pm \frac{1+2n}{n^2(n+1)^2}) + \frac{2e^{-1}}{n_c^2}(1\mp 1)\Big], \end{aligned} \tag{26}$$

where the subindex $\pm$ is for the even and odd contribution. Then eq. (26) gives the one-loop finite contribution to the 4D mass, with brane kinetic terms.

We can compare the finite 4D mass term with brane kinetic couplings with the one without them. To do this we consider that the 4D couplings of both theories are the same (for constant field $\phi_c$). Then we get

$$\begin{aligned} m^2_{4D\pm} &\to \frac{\lambda_{4D}}{16\pi^4 R^2}\frac{\zeta(3)\pm 1/2}{8}, & a\to 0, \\ m^2_{4D\pm} &\simeq \frac{\lambda_{4D}}{16\pi^4 R^2}[\frac{\zeta(3)-1/2}{8} + \frac{\alpha_\pm}{n_c^2} + O(1/n_c^3)], & a\to\infty, \end{aligned} \tag{27}$$

where $n_c = a/(2\pi R)$ and

$$\alpha_\pm = 3/2 - 4/e, 0. \tag{28}$$

In eq. (27) we have put the first correction in powers of $1/n_c$. The odd field doesn't coupled to the brane at $y=0$, that's why the mass doesn't change for this mode.

Now we compute the vertex with constant field $\phi_c$. First we discuss the high energy regime. As we said before, in this regime the field seems an odd field. Then we get the linear and logarithmic dependence on $\Lambda$ in the same way we did in the previous section.

Second, we consider the low energy regime that corresponds to windings $n > n_c$ in both propagators involved in the Feynman diagram. This case is the same as the one without brane terms, but summing over windings $n > n_c$. If the field is even under $Z_2$, there is an IR logarithm. Then, the finite and logarithmic one loop contribution to the 4D coupling are given by

$$\lambda^+_{4D} \simeq \frac{\lambda^2_{4D}}{16\pi^2}[C - \log(n_c R\mu_{ir}) - \frac{1}{2}\log(\Lambda R)], \tag{29}$$

where $C\sim -1$ and the first logarithm is the IR long distance. For an odd field we get a similar result without the IR logarithm.

Let's discuss now what happen with brane kinetic terms in $y_{fp} = \pi R$. In this case the limit of $a \gg R$ corresponds to an opaque brane, and the reflection has a minus sign. Then the effect of this brane is again the same as considering an odd field. So we can get an odd field puting (almost) opaque branes. This suggests a new way o symmetry breaking: let's suppose that the components of a multiplet have different brane couplings. Then the effect of these couplings will be the same as choosing different boundary conditions for the fields of a given multiplet, breaking the symmetry under which the multiplet transforms.

### 3.3 One-loop gauge coupling

We consider as an application of the previous formalism, a 5D theory with gauge fields and a scalar charged field, transforming with representation S. We want to get the logarithmic divergencies of the gauge coupling, due to the scalar fields, in an orbifold. Then we consider the one loop scalar contribution to the vacuum polarization $\Pi_{MN}$. This have been computed with K-K modes [12], here we get the same result with winding modes.

We consider the effective 4D theory, that is the theory obtained after integration over the extra dimension with constant fields (zero K-K modes). We define $g_0$ as the effective 4D coupling of an abelian theory, at the scale $\Lambda$. Then, after some manipulations, we can write the divergent part of the one-loop vacuum polarization as

$$\Pi(k^2=0) = \frac{g_0^2}{3}\sum_{q_5}\int\frac{d^4q}{(2\pi)^4}\frac{1}{(q_4^2+q_5^2)^2} = -\frac{g_0^2}{3}\int dy\int\frac{d^4p}{(2\pi)^4}\frac{d}{dp^2}\widetilde{G}^{orb}_{\pm}(p;y,y). \tag{30}$$

The momentum integral is a loop with two scalar propagators without external momenta, then it is the same as the one loop $\lambda$ contribution. On the r.h.s. of eq. (30) we have written the vacuum polarization in terms of one propagator, as we did for the scalar coupling.

The gauge coupling of the effective 4D theory is at the one loop level given by

$$g^{-2} = g_0^{-2}[1 + g_0^2\beta_0\log(\mu_{ir}R) - g_0^2\beta_1\log(\Lambda R)], \tag{31}$$

where

$$\frac{\beta_0}{2} = \beta_1 = \frac{1}{48\pi^2}. \tag{32}$$

In eq. (31), as in eq. (18) for the one-loop scalar coupling, the $\log(\Lambda R)$ comes from brane effects.

If the gauge group is non-abelian, then we only have to modify the charges and multiply by $t(S)$, where $tr[T_a(S)T_b(S)] = t(S)\delta_{ab}$.

Eq. (31) gives the scalar contribution to the 4D effective coupling at one loop, for a theory with a cut-off scale $\Lambda$. We can consider a theory with a different cut-off $\Lambda'$, and one-loop coupling $g'$. Then the relation between the couplings $g$ and $g'$ is given by

$$g' = g[1 + g_0^2\beta_1\log\frac{\Lambda'}{\Lambda}]. \tag{33}$$

# 4 6D winding renormalization

We apply winding modes formalism to a space with two extra-dimensions. Given an infinite plane $\mathcal{R}^2$ we can obtain a two-dimensional torus $T^2$ identifying with $G = \mathcal{Z}\times\mathcal{Z}$ ($\mathcal{Z}$ the integer numbers). The identification is $\bar{y}\sim\bar{y}+\bar{w}$, where $\bar{y} = (y^1, y^2)$ and $\bar{w} = 2\pi(n^1R_1, n^2R_2)$, $n^j\in\mathcal{Z}$. $R_1$ and $R_2$ meassure the size of the extra dimensions compactified in a torus, $y_j\in[0, 2\pi R_j)$. There is one parameter more to obtain a complete description of $T^2$, the angle betwen the directions of identification in the plane, as shown in fig. 4.

In the same way as in 5D we can find the scalar Green's function in euclidean infinite space, in mixed representation (momenta in Minkowski directions and coordinates in extra directions)

$$(p^2 - \partial^j\partial_j)\widetilde{G}(p;\bar{y}-\bar{y}') = \delta^{(2)}(\bar{y}-\bar{y}'), \tag{34}$$

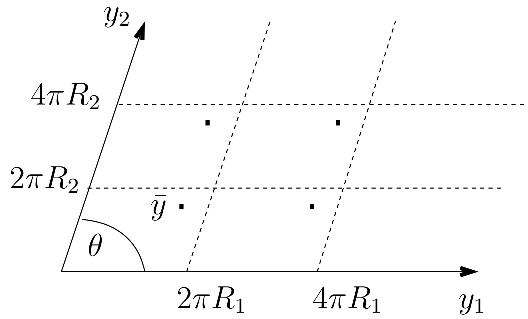



Figure 4: *Identification of $\mathcal{R}^2$ to obtain a bidimensional torus $T^2$, with parameters $\{R_1, R_2, \theta\}$.*

where $p^2 = p^\mu p_\mu$ is the 4D momentum and $j = 1, 2$ numbers the extra dimensions. The solution to this equation is

$$\widetilde{G}(p; \bar{y} - \bar{y}') = \frac{1}{2\pi} K_0(p|\bar{y} - \bar{y}'|), \tag{35}$$

where $K_0$ is $K$ Bessel function of zero order. Now we identify and get the propagator in the compact space

$$\widetilde{G}^{tor}(p; \bar{y}, \bar{y}') = \sum_{\bar{w}} \frac{K_0(p|\bar{y} - \bar{y}' + \bar{w}|)}{2\pi}, \tag{36}$$

with $|\bar{y}|$ the modulus of the vector messured with the flat metric of a torus

$$ds^2 = dy_1^2 + dy_2^2 + 2\cos^2\theta dy_1 dy_2. \tag{37}$$

As in the 5D case, the propagator for non-zero winding is exponentially damped at high energies $p|\bar{w}| \gg 1$

$$K_0(p|\bar{w}|) \to e^{-p|\bar{w}|}\sqrt{\frac{\pi}{2p|\bar{w}|}}, \tag{38}$$

showing that winding contributions will be always finite.

For arbitrarily small argument $K_0(x) \to -\log x$. Then the propagator diverges at short distances in the extra dimensions. Therefore to compute Feynman integrals with $\bar{y} \to 0$ we have to regulate the propagator[2] when $\bar{w} = 0$.

To compactify on an orbifold we have to introduce new identifications. A simple possibility is obtained introducing two $Z_2$ groups, one acting on $y^1$ and the other on $y^2$, in this way $y^j \sim -y^j$. Then, due two $\mathcal{Z}$ and $Z_2$ action on each direction, the fundamental domain becomes $[0, \pi R_j]$. We can act with each $Z_2$ independently, then we identify four different points on $\mathcal{R}^2$: $(y^1, y^2) \sim (\pm y^1, \pm y^2)$. According to this, the orbifold 6D propagator is

$$\begin{aligned}\widetilde{G}^{orb}(p; \bar{y}, \bar{y}') = \sum_{\bar{w}} [&\widetilde{G}(p; \bar{y} - \bar{y}' + \bar{w}) + p_1 \widetilde{G}(p; \bar{y} - \bar{x}' + \bar{w}) \\ &+ p_2 \widetilde{G}(p; \bar{y} + \bar{x}' + \bar{w}) + p_1 p_2 \widetilde{G}(p; \bar{y} + \bar{y}' + \bar{w})],\end{aligned} \tag{39}$$

[2] In D-dimensions with two of them in coordinate representation and D-2 in momentum representation, the propagator in flat infinite space is $\widetilde{G}^D(p; \bar{y}) = \int d^2 s \frac{e^{i\bar{s}.\bar{y}}}{p^2 + s^2}$, where we see that evaluating $\bar{y} = 0$ we get logarithmic divergences. We can regulate the short distance behaviour with an UV cut-off $|\bar{y}|_{min} = \Lambda^{-1}$, then the propagator becomes $\widetilde{G}^\Lambda(p; 0) = \pi \log(\frac{p^2 + \Lambda^2}{p^2})$.

where $\bar{x}' = (-y'^1, y'^2)$ and $p_j$ is the field parity in $j$ direction.

Here we can make the same analysis as in 5D: the new propagator terms will give localized divergent contributions. For zero winding the terms with just one coordinate identified under $Z_2$ will give 5D divergencies (localized in one extra dimension), and the term with both coordinates inverted will give 4D divergencies localized in a point of the extra space.

### 4.1 6D scalar renormalization on $T^2$

Using the winding modes we can easily separate cut-off dependent from finite contributions in 6D. To show this let's compute the radiative corrections of the scalar theory at one-loop with the extra space compactified on a torus $T^2$. The effective action is similar to eq. (6) but with two extra dimensions. For a torus $m^2(y)$ is constant, and is given by

$$m^2_{tor} = \frac{\lambda}{2} \int \frac{d^4p}{(2\pi)^4} \widetilde{G}(p, \bar{w}) = \frac{\lambda}{32\pi^3} \left(\frac{\pi^2\Lambda^4}{2} + \sum_{\bar{w}\neq 0} \frac{4}{|\bar{w}|^4}\right). \tag{40}$$

Again the divergent term is due to the zero winding mode and winding modes different from zero give the finite contributions.

Let's consider a constant field $\phi_c$. Integrating over the extra space we can compare the finite 4D masses obtained from a 5D theory with the ones obtained from a 6D theory. Making $\lambda_{5D} = \lambda_{6D}/2\pi R$ and summing over windings we get $m^{cir}_{4D} \simeq 1.5\ m^{tor}_{4D}$.

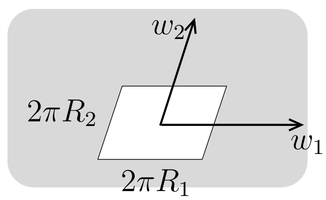



Figure 5: *We consider $\bar{w}$ a continuos variable, then it takes values over the plane. As we have separated the zero mode, we don't integrate over the parallelogram in the origin.*

Now we consider $S_4$ for a constant field $\phi_c$. Using the analog to equation (10) but with one more extra dimension we can write the 6D one-loop coupling as

$$\lambda_{tor} = -\frac{\lambda^2}{2} \sum_{\bar{w}} \int \frac{d^4p}{(2\pi)^4} \frac{d}{dp^2} \widetilde{G}(p; \bar{w}) = \frac{\lambda^2}{32\pi^3} \left(\frac{\Lambda^2}{4} + \sum_{\bar{w}\neq 0} \frac{1}{|\bar{w}|^2}\right), \qquad \phi = \phi_c. \tag{41}$$

The sum over modes is logarithmically divergent, in the same way as in 5D. We approximate the sum with an integral, then $\bar{w} \in R^2$. We have to exclude the zero mode, then the domain of intagration is shown in fig. 5. Therefore the second term of the r.h.s. of eq. (41) is

$$\simeq -\frac{\lambda^2}{16\pi^3 V_{tor}} [(\pi - \gamma)\log(\mu_{ir} R_1) + \gamma \log(\mu_{ir} R_2)], \quad \gamma = arctg\left(\frac{2R_1R_2\sin\theta}{R_1^2 - R_2^2}\right), \tag{42}$$

where $\mu_{ir}$ is an IR cut-off, the inverse of $|\bar{w}_{max}|$ and $V_{tor}$ is the torus volume.

## 4.2 6D scalar renormalization on an orbifold $T^2/Z_2 \times Z_2$

We repeat the steps done for the 6D torus, using the orbifold 6D propagator of eq. (39). For simplicity we consider a constant field $\phi_c$. Then we can write $S_2$ as

$$S_2 = -\phi_c^2\{m_{tor}^2 V_{orb} + m_f^2 + \frac{\lambda}{32\pi^3}[\sum_{j=1,2}\frac{p_j V_{orb}\Lambda^3}{6\pi R_j} + \frac{p_1p_2}{2}\Lambda^2\theta]\}, \qquad \phi = \phi_c \tag{43}$$

where $V_{orb}$ is the orbifold volume and $m_f^2$ is a finite term given by

$$\begin{aligned} m_f^2 =& \frac{\lambda}{32\pi^3}\{\sum_{j=1,2}\frac{-p_jV_{orb}}{6\pi^4R_j^4} - \frac{p_1p_2}{8\pi^2\sin^2\theta}[\frac{f(\beta,\theta-\beta)-\sin 2\theta}{R_1^2} - \frac{f(\theta-\beta,\beta)-\sin 2\theta}{R_2^2}]\} \\ &+ \frac{\lambda}{32\pi^3}\int d^2y\sum_{\bar{w}_{fin}}[\frac{p_1}{|2(y^1,0)+\bar{w}|^4} + \frac{p_2}{|2(0,y^2)+\bar{w}|^4} + \frac{p_1p_2}{|2\bar{y}+\bar{w}|^4}], \end{aligned} \tag{44}$$

the function $f(x,y)$ is defined by

$$f(x,y) = 2x + \sin 2y, \qquad \sin^2(\beta) = \frac{(R_2\sin\theta)^2}{(R_1)^2+(R_2)^2+2R_1R_2\cos\theta}. \tag{45}$$

The sum in eq. (44) is over windings that do not give divergencies. Then we have to exclude windings given by $(n^1, n^2) = (0,0), (-1,0), (0,-1), (-1,-1)$, when these windings give divergencies.

The $\Lambda^3$ divergencies in eq. (43) are localized in one direction, this can be seen considering fields $\phi(\bar{y})$, and expanding them in power series around the fixed points. This divergencies are similar to the bulk divergencies in 5D. Using the series expansion it can be seen that $\Lambda^2$ terms are 4D, they are localized on the four fixed points $\bar{y}_{fp}$. To obtain divergent localized kinetic terms in the directions of the extra dimensions, we have to consider the terms of second order in the series expansion.

Now we consider $S_4$ for the orbifold with constant $\phi_c$. Integrating (10) in 6D with the orbifold propagator we obtain $S_4$ at one-loop. We approximate the sum over windings with an integral. The result depends on the $Z_2$ parity $p_j$, and is given by

$$\begin{aligned} S_4 \simeq& \frac{\lambda^2\phi_c^4}{32\pi^3}\{\frac{\Lambda^2V_{orb}}{4} + \sum_{j=1,2}\frac{\Lambda p_j\pi R_j\sin\theta}{8} + 2p_1p_2[\beta\log(\Lambda R_1) + (\theta-\beta)\log(\Lambda R_2)] \\ &- (1+p_1+p_2+p_1p_2)[\frac{\pi-\gamma}{2}\log(\mu_{ir}R_1) + \frac{\gamma}{2}\log(\mu_{ir}R_2)]\}, \end{aligned} \tag{46}$$

where we have regularized the winding sum with an IR cut-off $\mu_{ir}$.

The IR logarithmic contributions are cancelled if the scalar field is odd in any of the directions. This again is easier to understand with K-K decomposition, there is zero mode just for the even-even case. Evaluating this equation for $\{R_1 = R_2 = R, \theta = \pi/2\}$ is very easy to compare the linear and logarithmic divergencies with the 5D case.

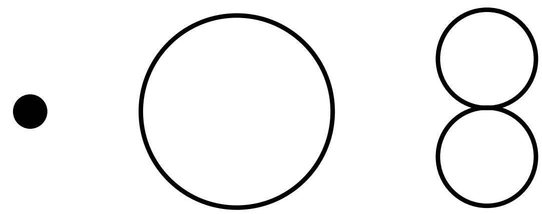
The radiative corrections show that we should include bulk, 5D and 4D localized masses from the begining, and also localized vertices and kinetic terms.

# 5 Radion stabilization in plane orbifolds

As an application of the winding formalism, we compute in this section the leading two loop contributions to the effective potential for the radion, in a product space $\mathcal{R}^4 \times S^1/Z_2$. We will see that under certain symmetry assumptions we can get a Coleman-Weinberg potential. Therefore the size of the extra dimension can be stabilized at large values.

## 5.1 Scalar potential

Let's consider a scalar 5D theory, as the one in section 3.1, compactified on an orbifold. The effective potential for the radion at tree level is zero, so we compute loops to obtain a sensible effective potential (see fig. 6) [3]. We calculate these quantum corrections using the winding formalism. Let's start with the one-loop term. In K-K modes we can write the one loop



(a) (b) (c)

Figure 6: *Perturbative expansion of the effective potential for the radion. The first diagram (a) is the tree level contribution, that cancels out.*

effective potential as

$$V^{(1)} = \sum_{k_5} \int \frac{d^4k}{(2\pi)^4} \log(k^2 + k_5^2). \tag{47}$$

To obtain it in winding representation we can write the last equation as

$$V^{(1)} = \sum_{k_5} \int \frac{d^4k}{(2\pi)^4} \int dk^2 \frac{1}{(k^2 + k_5^2)}. \tag{48}$$

The last factor is the scalar propagator in K-K modes, then we can replace it by the one with winding modes and integrate

$$\begin{aligned} V^{(1)} &= \sum_n \int \frac{d^4k}{(2\pi)^4} \int dk^2 \int_0^{\pi R} dy\, [\widetilde{G}(k, 2n\pi R) \pm \widetilde{G}(k, 2y + 2n\pi R)] \\ &= \frac{1}{8\pi^2} [\frac{\Lambda^5}{5}\pi R - \frac{3\zeta(5)}{8\pi^4 R^4} \mp \frac{\Lambda^4}{16}], \end{aligned} \tag{49}$$

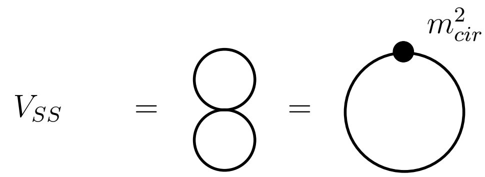
where $\widetilde{G}$ is defined in eq. (2), $\zeta$ is the Riemmann zeta function ($\zeta(\alpha) = \sum_{n>0} 1/n^\alpha$), and we get the finite term from no zero windings.

[3] If the scalar vev $\langle\phi\rangle \neq 0$, we also have to include a two loop diagram, with two three-point vertices, each of them proportional to the vev [13].

After that we compute the two loop term $V^{(2)}$, shown in (c) of fig. 6. It is given by

$$V^{(2)} = \frac{\lambda}{2}\sum_{n,n'}\int_0^{\pi R} dy \int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}[\widetilde{G}(k,2n\pi R) \pm \widetilde{G}(k,2y+2n\pi R)] \\ \times[\widetilde{G}(q,2n'\pi R) \pm \widetilde{G}(q,2y+2n'\pi R)]. \tag{50}$$

Once more zero winding modes are divergent and non zero modes give the finite contributions. If we call $\widetilde{G}_S$ the first term of the orbifold propagator (identic to the circle propagator) and $\widetilde{G}_Z$ the second term (obtained with $Z_2$ identification), then we can write equation (50) as $V^{(2)} = V_{SS} \pm 2V_{SZ} + V_{ZZ}$. Let's consider first the term $V_{SS}$. Every loop is similar to the one loop contribution to the mass. Then $V_{SS}$ of eq. (50), can be interpreted as one loop with a massless propagator $\widetilde{G}_S$ and a mass $m_{cir}^2 \propto \lambda[\Lambda^3/6+\zeta(3)/(8\pi^3R^3)]$, as is shown in the following Feynman diagram




The mass $m_{cir}$ is of order $\Lambda$, then it can not be considered a perturbation. Therefore we have to consider terms with arbitrary number of mass insertions, as is shown in fig. 7. If we sum

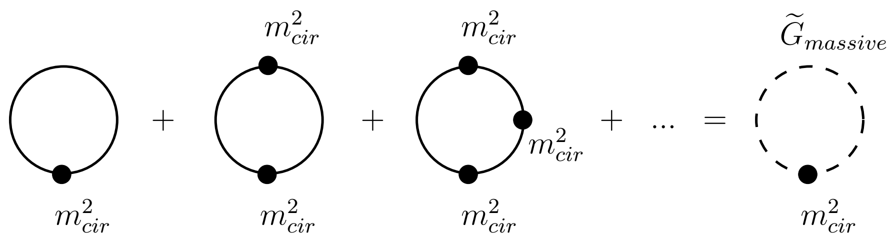



Figure 7: *Feynman diagrams giving the one loop effective potential with mass insertions. The mass $m_{cir}$ itself is one loop, it is given by the one loop contribution to the mass. The continuos lines are for masless propagators and the dashed line is for the massive one.*

the series of fig. 7, we obtain a loop with a massive propagator and a mass vertex. As we discussed in section 2, a massive propagator is obtained replacing $p \to \sqrt{p^2+m_{cir}^2}$. If the mass is large, $m_{cir} \gg R^{-1}$ (as is the case because $m_{cir} \sim \Lambda \gg R^{-1}$), we can approximate the propagator by $e^{-m_{cir}|y+2n\pi R|}/m_{cir}$. In this case the propagator is exponentially damped and cancels before making windings. Then the only relevant contributions are divergent, and $V_{SS} \simeq 2\lambda\Lambda^6\pi R/(96\pi^2)^2$.

If there is a symmetry prohibiting divergent masses, as a local gauge symmetry, then the finite contributions are given by $V_{SS} \simeq 2\lambda\zeta(3)^2/(128^2\pi^9R^5)$.

We can make a similar analysis for the other topologies, obtaining the same results. Summing over topologies we get

$$V^{(2)} = \frac{\lambda}{64\pi^4}[A\,(\pi R)\Lambda^6 + B\Lambda^5 + C(\pi R)^{-5}], \tag{51}$$

where $A \sim 10^{-1}, B \sim 10^{-1}, C \sim 10^{-2}$, and the sign of $B$ depends on field parity under $Z_2$. If there is no symmetry protecting $V$ from divergencies, the divergent terms are dominant, in the other case we only get the $R^{-5}$ finite term.

## 5.2 Effect of brane kinetic terms

We want to obtain a potential able to stabilize a large extra dimension. Then we consider new brane terms: we add to the previous set-up fermion fields localized on the fixed points. If these 4D fields couple to the bulk ones, there are new contributions to the two loop effective potential. There is a new term with a fermionic-loop localized on the branes and a scalar-loop on the bulk, as is shown in fig. 8. To be more precise we consider the following interaction

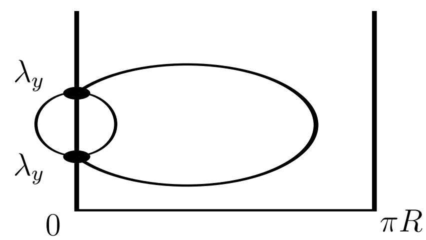



Figure 8: *Feynman diagram with one localized fermionic loop and one bulk scalar loop, contributing to the effective potential.*

$\mathcal{L}_y = \delta(y)\lambda_y \phi\bar{\psi}\psi$, localized on the brane at $y_{fp} = 0$, and compute the two loop contribution.

The 4D fermionic loop has several terms given by

$$\frac{\lambda_y^2}{8\pi^2}[-2\Lambda^2 + \frac{5}{3}q^2 - q^2 \log \frac{q^2}{\Lambda^2}]. \tag{52}$$

We are interested in the logarithmic kinetic contributions. The first and second terms are local, but the third is not local and will give us new things.

The term of the scalar effective action that couples to the brane loop is

$$\frac{\lambda_y^2}{8\pi^2} \int_0^{\pi R} dy \int_0^{\pi R} dy' \int \frac{d^4p}{(2\pi)^4} \delta(y)\delta(y')\phi(y)p^2 \log(\frac{p^2}{\Lambda^2})\phi(y'). \tag{53}$$

Then, the two loop contribution with a loop localized on one of the branes, is given by

$$\begin{aligned} V_b^{(2)} &= \frac{\lambda_y^2}{8\pi^2} \sum_n \int \frac{d^4p}{(2\pi)^4} \widetilde{G}(p; 2n\pi R) p^2 \log(\frac{p^2}{\Lambda^2}) \\ &= \frac{\lambda_y^2}{64\pi^4}[\frac{\Lambda^5}{25} + \sum_{n\neq 0} \frac{50 - 24\gamma - 24\log(2n\pi R\Lambda)}{(2n\pi R)^5}], \end{aligned} \tag{54}$$

as usual, the divergent term is due to the zero winding.

Now we have to sum $V^{(1)} + V^{(2)} + V_b^{(2)}$ to get the effective potential to two loops. Let's suppose that for each boson there is a fermion with equal boundary conditions. Then the finite

part of $V^{(1)}$ cancels out, because fermionic loops have a minus sign. Furthermore, if there is a local symmetry (like local supersymmetry) protecting the effective potential from divergent terms, then only the finite terms in $V^{(2)}$ and $V_b^{(2)}$ remain, and the two-loop effective potential is given by

$$V \sim \frac{1}{R^5}[cte. - \log(\Lambda R)], \qquad cte. \sim 1. \tag{55}$$

This is a Coleman-Weinberg potential [14]. We know that this kind of potential can stabilize $R$ with a large value $R \gg \Lambda^{-1}$, that is consistent with our renormalization assumptions and the large extra dimensions scenario. In this way quantum corrections in a product space with matter fields localized on the branes can stabilize the size of the extra space, with large radius.

# 6 Conclusions

We have used the winding formalism to compute radiative corrections on a theory with extra dimensions. It allows us to separate, in a very clear and intuitive way, cut-off dependent from finite corrections. It is also very easy to see how the brane terms are generated at one loop. We extended this formalism to a higher dimensional space, and showed that it is immediate to separate finite from cut-off dependent contributions, making this method very useful.

We explored the effects of parallel and perpendicular kinetic brane terms. Our conclusions are that, whenever there are perpendicular kinetic terms on the branes, the theory can be redefined, in such a way that there only remain parallel kinetic terms. We also argued that brane kinetic terms can provide a new way of symmetry breaking.

We applied the winding formalism to compute the finite terms of scalar masses, in theories with one and two extra dimensions. If there is a symmetry (like supersymmetry, or other global or gauge symmetries) that protects masses from divergencies, this finite terms are predictions of the theory. We also get, in theories with one and two extra dimensions, the logarithmic contributions to the 4D couplings in a simple way.

We analyzed the possibility of getting a potential stabilizing the size of the extra space, when the higher dimensional space can be approximated by a direct product. We saw that it is very simple to compute the two loop effective potential with winding modes. We also showed an scenario with brane terms and bulk fields where the extra volume can be stabilized with a Coleman-Weinberg potential.

# 7 Aknowledgments

I would like to thank Álex Pomarol for many ideas, discussions and constant advise. I also acknowledge fruitful discussions with A. Flachi and O. Pujolàs. This work was supported by the Spanish Education Office (MECD) under an FPU scolarship.

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